The Inability to Collapse Three Dimensions Into Two Dimensions Physically

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Abstract

This investigation evaluates the physical viability of a literal, mathematical dimensional reduction within the universe, specifically addressing the question: "Can a three-dimensional world create a pure two-dimensional plane?" While lower-dimensional spaces serve as effective mathematical models, forcing a spatial coordinate metric to compress entirely to absolute zero breaches the foundational boundaries of classical and modern physics. This inquiry evaluates the structural breakdown across three physical frameworks. First, macroscopic geometric metrics analyzed through epsilon-delta limits and Lebesgue measures prove that collapsing the z-axis boundary to a singular region ($z = a$) reduces three-dimensional volume to exactly zero, forcing physical density to diverge to infinity or necessitating a non-physical mass of zero. Second, evaluating a constrained quantum system through the time-independent Schrödinger equation shows that as the confinement wall length approaches zero ($L_z \to 0$), the localized energy states diverge to infinity ($\lim_{L_z \to 0} E_z = \infty$), demanding infinite energy or collapsing the system into a gravitational singularity. Finally, evaluating Einstein's field equations in general relativity under these metric constraints transforms the energy density component (T_{00}) of the stress-energy tensor into an impossible zero-denominator fraction. This forces the tensor to blow up to infinity ($T_{\mu\nu} = \infty$), yielding an infinite local curvature of spacetime ($G_{\mu\nu} = \infty$). Ultimately, these multi-disciplinary mathematical failures confirm that a pure two-dimensional plane cannot physically exist as a structural product of a three-dimensional universe.

Keywords

Dimensional Reduction · Mathematical Physics · Quantum Mechanics · General Relativity · Lebesgue Measure

1 Introduction

Humans—us, living in a world where laws seem trivial, space seems like a given, and our three-dimensional universe is not second-guessed—rarely question the fabric of our reality. But now let's second guess this world, our world, containing three distinct spatial dimensions on a plane that govern everything around us, (x, y, z). They allow for mass, positions, shapes, and so much more. Now what happens if you take one of those dimensions away, you are left with (x, y). A two dimensional plane. So what? Two-dimensional planes appear in a variety of topics, even everyday life. What if we were to take a closer look at it, what a two dimensional plane really is, could it even exist physically? Of course it can, something so simple wouldn't have a problem in its own plane, even if the rules governing that plane were different, Or would it exist?

This paper will proceed to go in depth to force an answer out of the question, "Can our Three Dimensional World Create a Two Dimensional Plane?". First, the spatial dimensions must all be defined correctly and accurately, both mathematically and linguistically. In this proof, the physical conditions will be extended to infinity from all directions, even with the knowledge that the observable universe is not infinite. This is because, as it will become clear, it is irrelevant towards the outcome or answer whether or not we incorporate infinity. Three main paths will be taken to find an answer to this question, our macroscopic world, quantum world, and through general relativity.

To arrive at an answer, this study approaches the problem from three different physical angles. First, we analyze macroscopic geometric constraints regarding mass and volume. Second, we dive into the quantum world examining different energy states through the time independent Schrödinger equation, with constraints. Finally, we investigate Einstein's field equations in General Relativity, testing how the stress-energy tensor behaves when a spatial dimension collapses to zero.

2 Macroscopic Physical Dimensions

2.1 Three Dimensions to Two Dimensions Geometrically - Volume : Area

In our world, we have three distinct spatial dimensions, noted by the coordinates (x, y, z). Furthermore, everything in our world has length, width, and height, also implying volume, generally given by formulas relating the three, $V = \Delta x \Delta y \Delta z$. In a more arbitrary way to display volume, we can use triple integrals,

$$V = \iiint_R dV,$$

where R is the region or shape of which we are finding the volume of. Lets assume we are comprised of two dimensional slices. For this to be true, we

would have to be made of infinitely many slices of zero height. Or represented mathematically as, $\infty \times 0 = 0$. In measure theory infinity times zero is zero, which we will be using in this paper multiple times. Now, we can define all of three dimensional space as,

$$V = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} 1 \; dz \; dy \; dx = \infty.$$

Note, our bounds to the observable universe are not infinity. However, in our calculations and proof, there is no difference physically or mathematically between the two. If we work out all the math using Hubble's constant and universe growth, we will ultimately result in the same answer. So for simplicity, we use infinity. To transition this from three dimensions to two dimensions, we must define two dimensional space, then transition from three to two by eliminating one of the spatial positions. The first way we could do this by using limits to evaluate the volume as the change in z approaches zero, $\Delta z \to 0$. We have,

$$\lim_{\Delta z \to 0} \Delta x \Delta y \Delta z = 0.$$

We can prove this limit exist using the Epsilon-Delta definition of the limit, which states,

$$\lim_{x \to a} f(x) = L \quad \text{if} \quad \forall \varepsilon > 0, \; \exists \delta > 0 \text{ such that if } 0 < |x - a| < \delta, \text{ then } |f(x) - L| < \varepsilon.$$

To prove our limit we start with wanting to find a $\delta > 0$, such that if $0 < |\Delta z| < \delta$, then $|\Delta x \Delta y \Delta z| < \varepsilon$. Isolating Δz in our epsilon inequality gives,

$$\Delta z < \frac{\varepsilon}{|\Delta x||\Delta y|}.$$

We assume $|\Delta z| < \delta$, substituting this gives

$$\delta = \frac{\varepsilon}{|\Delta x||\Delta y|},$$

which is our chosen delta value. Now the formal proof, assuming $\Delta x \neq 0$ and $\Delta y \neq 0$, the product $|\Delta x \Delta y| > 0$. We suppose $0 < |\Delta z| < \delta$. Evaluating the absolute value of the function, $|\Delta x \Delta y \Delta z - 0| = |\Delta x \Delta y||\Delta z|$, then substituting the inequality $|\Delta z| < \delta$. $|\Delta x \Delta y \Delta z - 0| < |\Delta x \Delta y| \cdot \delta$. Then substituting our chosen delta value,

$$|\Delta x \Delta y| \left(\frac{\varepsilon}{|\Delta x \Delta y|} \right) = \varepsilon.$$

Therefore, whenever $0 < |\Delta z| < \delta$, $|\Delta x \Delta y \Delta z - 0| < \varepsilon$. Proving the limit exists,

$$\lim_{\Delta z \to 0} \Delta x \Delta y \Delta z = 0.$$

Hence, as the z-axis approaches zero such as in a true two dimensional plane, the entire two dimensional plane collapses to zero as well. Backing the claim that a three dimensional plane cannot be compressed into two dimensions.

The second way to complete this is more accurate, and it is to define all of two dimensional space as,

$$A = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} 1 \; dx \; dy = \infty,$$

A = area. We aim to remove the z spatial position. We can do this by using the triple integral representation of volume then collapsing the z-axis bounds to one region, $z = a$. This gives the integral of

$$V = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{a}^{a} 1 \; dz \; dy \; dx = 0.$$

It equals zero as the z-axis collapses to zero both mathematically and physically, meaning it equals zero, and $A = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} 0 \; dx \; dy = 0$. This pushes the statement that you cannot physically transition from a three dimensional plane to a two dimensional plane. We can prove this is true using measure theory, specifically Lebesgue measures and Lebesgue integrals (Folland 1999).

Let λ_3 be the 3-dimensional Lebesgue measure on $\mathbb{R}^3$, and let E be the subset of $\mathbb{R}^3$ defined by the plane $z = a$, $E \subset \mathbb{R}^3$.

$$E = \left\{ (x, y, z) \in \mathbb{R}^3 | x \in (-\infty, \infty), y \in (-\infty, \infty), z = a \right\}.$$

The volume, or Lebesgue measure of E is zero.

$$V = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{a}^{a} 1 \; dz \; dy \; dx = 0.$$

The indicator function of the set E, is denoted by $1_E(x, y, z)$, is a non-negative measurable function. The volume of E is its Lebesgue measure $\lambda_3(E)$, which can be expressed as a Lebesgue integral,

$$V = \int_{\mathbb{R}^3} 1_E \; d\lambda_3.$$

Since the integrand is non-negative, we apply the Fubini-Tonelli Theorem to write the multidimensional Lebesgue integral as,

$$V = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} 1_{\{a\}}(z) \; d\lambda_1(z) \right) d\lambda_1(y) \right) d\lambda_1(x),$$

decomposed into it's product measure, $\lambda_3 = \lambda_1 \times \lambda_1 \times \lambda_1$. To evaluate the innermost integral with respect to z, the set $\{a\}$ which is a singleton set in $\mathbb{R}$. A property of the one dimensional Lebesgue measure λ_1 is that any singleton set has the measure of zero.

$$\lambda_1(\{a\}) = a - a = 0.$$

Hence, the innermost integral evaluates to zero,

$$\int_{-\infty}^{\infty} 1_{\{a\}}(z)\ d\lambda_1(z) = \int_a^a 1\ dz = 0.$$

Substituting this back into our multidimensional Lebesgue integral yields,

$$V = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 0\ dy\ dx = 0.$$

Thus, the three dimensional volume of a two dimensional plane, $z = a$, is exactly zero, proving transitioning between the two physically results in zero. These two conclusions prove that volume cannot exist in two dimensional space, as well as transitioning through three and two dimensional space is not possible. The thought that volume cannot exist in two dimensional space will become interlaced with mass in the following subsection, and be referenced again in section three and four.

2.2 Mass, Density, and Volume in The Transition of Three to Two Dimensions

Since we have defined volume cannot exist in two dimensional space, and transitioning between them physically equals zero, we now take a look at how it relates to mass and density. A fundamental formula in physics that relates all three is $\rho = \frac{m}{V} \iff V = \frac{m}{\rho} \iff m = V \times \rho$. Now we substitute our given volume in two dimensions into each of these equations to note how the behave. Substituting volume into the first equation, $\rho = \frac{m}{0} = \pm\infty$, the second equation, $0 = \frac{m}{\rho}, m = 0, \rho \neq 0$, otherwise it goes to $\pm\infty$, and the third equation, $m = 0 \times \rho, m = 0$. These equations all either result in a mass of zero, or diverge to infinity. Hence in a two dimensional plane that was transitioned down from three, mass must be equal to zero, $m = 0$.

3 Quantum Physical Dimensions Through Schrodinger Equation

Our quantum world behaves differently compared to our macroscopic world, through different classifications and frameworks. The main component is the quantum wavefunction, Ψ, as it contains the information about our quantum system. For this part of the proof, we will utilize the Schrodinger equation and different energy levels of particles.

3.1 Schrodinger Equation in The z-axis

For our example, we will define a particle in the z-axis with the wavefunction, Ψ_z. We can model the Schrodinger equation for the particle in the z-axis as,

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_z}{dz^2} + V(z)\Psi_z = E_z\Psi_z. \tag{1}$$

We are going to set $V(z) = 0$, for a free particle. Giving the equation,

$$\frac{d^2\Psi_z}{dz^2} = \frac{-2mE_z}{\hbar^2}\Psi_z \implies \frac{d^2\Psi_z}{dz^2} + \frac{2mE_z}{\hbar^2}\Psi_z = 0. \tag{2}$$

We then set,

$$\kappa = \frac{\sqrt{2mE_z}}{\hbar} \iff \kappa^2 = \frac{2mE_z}{\hbar^2}.$$

This transforms our Schrödinger equation to

$$\frac{d^2\Psi_z}{dz^2} + \kappa^2\Psi_z = 0,$$

and the solutions to the Schrodinger equations are,

$$\Psi_z = A\sin(\kappa z) + B\cos(\kappa z).$$

3.2 Boundary Conditions and Energy Level in The z-axis

We set our boundary conditions to those of the particle in a box, meaning, $\Psi_z(0) = 0$ and $\Psi_z(L_z) = 0$ (Griffiths and Schroeter 2018). Substituting these values in and solving gives,

$$\Psi_z(0) = A\sin(\kappa(0)) + B\cos(\kappa(0)) = 0 \implies B = 0.$$

$$\Psi_z(L_z) = A\sin(\kappa L_z) = 0 \implies \sin(\kappa L_z) = 0.$$

Values of the inner product of $\sin(\kappa L_z)$, must have the value of $n_z\pi : n_z \in \mathbb{N} > 0$ to be equal to zero. We can set our inner product to this value, as well as substituting the value of κ back in, yielding,

$$n_z\pi = L_z\frac{\sqrt{2mE_z}}{\hbar}.$$

Solving for our energy level in the z-axis,

$$E_z = \frac{\hbar^2 n_z^2\pi^2}{2mL_z^2}.$$

3.3 Three Dimensions - Connecting to Two Dimensions

Since we have derived our energy level in the z-axis, we can take the limit as $L_z \to 0$. simulating a three dimensional plane collapsing into two dimensions by removing the third spatial position, z.

$$\lim_{L_z\to 0} E_z = \lim_{L_z\to 0} \frac{\hbar^2 n_z^2\pi^2}{2mL_z^2} = \infty. \tag{3}$$

Furthermore, prior to connecting this to three dimensional energies within the Schrodinger equation, converting to two dimensions from three. We can prove this limit using the M-Delta definition of the limit, which states,

$$\forall M > 0, \exists\ \delta > 0 \quad \text{such that} \quad 0 < L_z < \delta \implies E_z > M.$$

Starting with the scratch work, we want to force $E_z > M$, meaning we must choose a strict upper bound for L_z. We set up the inequality,

$$\frac{\hbar^2 n_z^2 \pi^2}{2mL_z^2} > M.$$

Since $L_z > 0 \quad \text{and} \quad M > 0$, we take the reciprocal of both sides, yielding,

$$\frac{2mL_z^2}{\hbar^2 n_z^2 \pi^2} < \frac{1}{M}.$$

Isolating L_z,

$$L_z < \frac{\hbar n_z \pi}{\sqrt{2mM}}.$$

This tells us our delta value,

$$\delta = \frac{\hbar n_z \pi}{\sqrt{2mM}}.$$

Now constructing the formal proof. If

$$E_z = \frac{\hbar^2 n_z^2 \pi^2}{2mL_z^2}$$

where $\frac{\hbar^2 n_z^2 \pi^2}{2mL_z^2} > 0$, then $\lim_{L_z \to 0} \ E_z = \infty$. Let $M > 0$ be given. Choose

$$\delta = \frac{\hbar n_z \pi}{\sqrt{2mM}}.$$

Assume $0 < L_z < \delta$. We must show that this implies $E_z > M$. We start with,

$$L_z < \frac{\hbar n_z \pi}{\sqrt{2mM}},$$

squaring both sides,

$$L_z^2 < \frac{\hbar^2 n_z^2 \pi^2}{2mM},$$

isolating M,

$$M < \frac{\hbar^2 n_z^2 \pi^2}{2mL_z^2}.$$

Substituting back our definition of E_z, so $M < E_z \iff E_z > M$. Therefore, because $0 < L_z < \delta \implies E_z > M$, we have satisfied the formal definition, therefore,

$$\lim_{L_z \to 0} \ \frac{\hbar^2 n_z^2 \pi^2}{2mL_z^2} = \infty.$$

Hence, as our wall length or z spatial metric approaches zero, our energy level diverges to infinity. This cannot physically happen as it would collapse into a singularity. We can utilize this by expanding our energy to three dimensions,

$$E = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right), \tag{4}$$

$E = E_x + E_y + E_z$. To remove the third spatial dimension, we can find the limit as $L_z \to 0$. This will then give us our energy value in two spatial dimensions as the third one would have collapsed.

$$\lim_{L_z\to 0} E = \lim_{L_z\to 0} \frac{\pi^2\hbar^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \infty, n_x, n_y, n_z \in \mathbb{N} > 0 \tag{5}$$

We can prove this limit with the definition, $\forall M > 0, \exists\ \delta > 0 \quad \text{such that} \quad 0 < L_z < \delta \implies E > M$. Since completing all the algebraic steps would be repetitive, we will leave it out. However, the limit has been proved. This concludes that transforming a three dimensional quantum system to a two dimensional quantum system would require an infinite amount of energy, which is physically impossible.

$$\lim_{L_z\to 0} \frac{\pi^2\hbar^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \infty,$$

meaning a quantum system converted from three spatial dimensions to two spatial dimensions would collapse into a singularity, or require infinity energy to successfully achieve. That is also ignoring the mass calculations, which will also blow it up to infinity since mass equals zero in the converting three dimensional space to two dimensional space. This can be shown by the limit,

$$\lim_{m\to 0} \frac{\pi^2\hbar^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \lim_{m\to 0} \frac{\pi^2\hbar^2}{0}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \infty,$$

which can also be proved using the M-Delta method, and was proved outside of this paper. Keep in mind, that the mass is only zero in this specific case because we are analyzing the transition between three spatial dimensions to two, where mass is equal to zero. Not whether or not mass could exist in two dimensions (Ando, Fowler, and Stern 1982).

4 General Relativity Physical Dimensions

4.1 Connection to the Macroscopic World

General relativity bridges the gap between relativistic geometry and our macroscopic world by defining how physical properties dictate the structure of our universe. General relativity is the branch of physics that explains spacetime and its dynamic properties, specifically through the interactions of mass and

gravity. This framework is governed by a fundamental set of field equations derived by Albert Einstein in 1915:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{6}$$

which dictates the warping and curvature of spacetime (Carroll 2019).

In our macroscopic world, gravity is typically viewed as a simple Newtonian force acting on mass, $\vec{F}_g = mg$. However, Einstein's field equations depict that spacetime depends on how mass is distributed across a given volume, a relationship tied between mass density, momentum, and internal pressure. These macroscopic properties are on the right-hand side of the equation within the stress-energy tensor, $T_{\mu\nu}$. Since curvature is driven by energy density rather than mass alone, compressing a fixed mass into a smaller volume increases the density, bending the surrounding space and time far more intensely.

Consequently, the physical presence of mass and energy is the makeup of gravitational fields. Einstein's equations dictate that a mass of zero, would cause the stress-energy tensor to vanish, ($T_{\mu\nu} = 0$). From a macroscopic perspective, this implies that without mass or energy density to maintain a physical presence, there is nothing able to warp the local spacetime. Under these conditions, the field equations reduce to a vacuum state, confirming that empty space remains intrinsically flat and un-warped unless acted upon by distant gravitational sources or a non-zero cosmological constant.

4.2 Stress Energy Tensor and Einstein Field Equations

An important part of Einstein's field equation that will be the focus of this subsection is the stress energy tensor,

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}. \tag{7}$$

The top rows, columns 1-3, represent the momentum density moving in spatial directions, (x, y, z). The left column, rows 1-3 is the energy flux, the flow of energy through a surface area per unit time. The bottom components represent the forces acting on the boundaries of a volume element. The diagonal elements are isotropic pressure pushing outward or pulling inward. The off-diagonal elements represent shear stress, which describe viscous forces sliding parallel to the surface.

However, we will be focusing on T_{00}, which represents $\frac{\text{Energy}}{\text{Spatial Volume}}$. Now forcing the stress energy tensor into two dimensions forces $V = 0$, so, $T_{00} = \frac{\text{Energy}}{0} = \infty$. This is because we previously stated that volume cannot exist in two dimensions.

This affects the rest of the stress energy tensor by transforming it into,

$$T_{\mu\nu} = \begin{bmatrix} \infty & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix} = \infty, \tag{8}$$

regardless of the other values. Analyzing Einstein's field equations when held to these constraints yields,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \implies G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} \infty \implies G_{\mu\nu} = \infty.$$

By Einstein's model and equations, this would mean the local curvature of spacetime is infinite, which we know is false. Since by principal, Einstein's field equations must work in all dimensions, as they describe the fabric of spacetime and reality itself. Yet they fail when compressing three dimensional space into two dimensional space.

5 Conclusion

In this paper, we went in depth to force an answer out of the question, "Can our Three Dimensional World Create a Two Dimensional Plane?". By taking three main paths through our macroscopic world, the quantum world, and general relativity, we have shown that physically collapsing three spatial dimensions into two is impossible. The rules governing our universe simply do not allow a spatial dimension to be stripped away without completely breaking the physics behind it.

First, looking at macroscopic geometric constraints, we proved through both limits and Lebesgue measures that collapsing the z-axis bounds to a single region ($z = a$) forces the three-dimensional volume to exactly zero. When we interlace this zero-volume constraint with mass and density, the fundamental equations either require mass to equal zero ($m = 0$) or cause the density to diverge to $\pm\infty$. This geometrically backs the claim that a three-dimensional plane cannot physically compress into two dimensions while maintaining a physical presence.

Second, diving into the quantum world through the time-independent Schrödinger equation, we modeled a particle in the z-axis and evaluated its energy levels. By taking the limit as the wall length approaches zero ($L_z \to 0$), the energy level diverges to infinity ($\lim_{L_z \to 0} E_z = \infty$). This means that transitioning a quantum system from three dimensions to two dimensions would either require an infinite amount of energy to successfully achieve or would immediately cause the system to collapse into a singularity. This breakdown is further compounded when factoring in mass, which similarly blows the energy equations up to infinity as mass approaches zero.

Finally, we bridged these concepts to the fabric of spacetime through general relativity and Einstein's field equations. Because volume cannot exist in a true

two-dimensional space transitioned down from three, forcing these constraints onto the stress-energy tensor turns the energy density component (T_{00}) into an impossible fraction divided by zero. This forces the entire stress-energy tensor to blow up to infinity ($T_{\mu\nu} = \infty$), which yields an infinite local curvature of spacetime ($G_{\mu\nu} = \infty$). While Einstein's equations must work in all dimensions by principle, they completely fail when compressing our existing spatial dimensions.

Ultimately, while two-dimensional planes are useful mathematical concepts in everyday life and theoretical topics, they cannot exist physically as a product of our three-dimensional world. Whether analyzing the macroscopic distribution of mass, the energy states of quantum wavefunctions, or the geometric warping of spacetime, the conclusion remains the same. Our three-dimensional universe cannot be collapsed into two dimensions; attempting to take one of those spatial dimensions away breaks the physical equations entirely, resulting in zeroes, infinities, and physical impossibilities.

6 References

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